Call for more rigorous research on savant syndrome

Makoto Yamaguchi

University of Tokyo, Japan

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Corresponding author:
Makoto Yamaguchi
University of Tokyo
Tokyo, Japan
E-mail: yamag-psy@toki.waseda.jp

Research on savant syndrome is fascinating. Some otherwise retarded individuals, often autistic people, show extraordinary feats; e.g., music, calculation, and drawing. Unfortunately, many reports on savant syndrome lack rigor, and some were even questioned about their credibility. I have pointed out such flaws before [1-3], thus there is no need to repeat them here.

Pointing out flaws in other researchers' reports is not pleasant at all. Unfortunately, however, without doing it, wrong facts are continuously read and cited by researchers, which is also impermissible. Here I must point out several additional things.

The late Hermelin [4], a leading researcher, attempted to explain mathematical facts about prime numbers. Unfortunately, it is extremely inaccurate. According to her, Fermat discovered the "prime number theorem", which shows that every prime number is either in the form $4n + 1$ or $4n – 1$, and Euler later formally proved it. (The smallest prime number 2 is an exception to this rule, so we must say *odd* prime numbers). She confirmed that this rule holds for 13 and 19, and said "Simple, isn't it?" In fact, it is really far too simple. Even lay readers may feel something is strange. As $4n$ and $4n + 2$ are even numbers, odd primes must of course be either $4n + 1$ or $4n – 1$. Such an easily seen statement cannot be called a theorem, and proving it does not need to wait for a genius like Euler. What she should have meant is *Fermat's theorem on sums of two squares*, which was indeed proved by Euler. The theorem is:

- An odd prime is expressed as the sum of two squares of integers if it is in the form $4n + 1$. (And an odd prime is not expressed as such if it is $4n – 1$. This fact is far easier to prove and was known from earlier days, so it is sometimes omitted from the theorem).

Her mistake was caused by inaccurate copying of the best-selling popular book Fermat's Last Theorem by Singh [5]. He stated that prime numbers are either $4n + 1$ or $4n – 1$, which she copied, and then soon after it (in the same paragraph) he (almost) correctly explained Fermat's theorem on sums of two squares. In sum, she copied a wrong part. Note that as Singh was trained not in mathematics but in physics, his descriptions are not entirely accurate. In fact, "the prime number theorem" means a different theorem. Perhaps Singh confused this Fermat's and Dirichlet's theorem, from which it can be derived that there are infinite numbers of both $4n + 1$ and $4n – 1$ type primes.

Next Hermelin explains Goldbach's conjecture inaccurately. The correct conjecture is:

- Every even number larger than 2 can be written as the sum of 2 prime numbers.

Therefore, her stating "larger than 24" is wrong, which is perhaps easily verified; e.g., 8 = 3 + 5. The cause of this mistake is unknown. Incidentally, she treats modular arithmetic as if it is evidence of modularity of the mind. Modular arithmetic has nothing to do with it.

Also troubling is her and her colleagues' methodology [6-8]. They compared a savant (Michael) with a single control subject. Comparison with one subject is usually not informative. The control subject seemed to have used trial division, but its details should have been clearly presented. Most people, even with substantial mathematical training, do not know that division is necessary only until $\sqrt{N}$, not $N/2$. This is because number theory is relatively isolated from other mathematical areas and those other areas are more emphasized in higher education. In addition, their presentation of data lacks rigor. Significant digits in the data were inconsistent.

Matthysse and Greenberg [8] explained basic modular arithmetic, and discussed the Fermat test and Carmichael numbers. (For the concrete method of the Fermat test using a spreadsheet, see [3]). Carmichael numbers are composite numbers that erroneously pass the Fermat test. They take up 561, the smallest Carmichael number, and state that $2^{560} \equiv 1 \bmod 561$, $3^{560} \equiv 1 \bmod 561$, $4^{560} \equiv 1 \bmod 561$, and so on. This is wrong. Popular mathematical books may simply state that Carmichael numbers always behave as if they were prime numbers in the Fermat test. However, more rigorous mathematical textbooks never fail to mention that Carmichael numbers mimic prime numbers unless the base is not relatively prime to that number. As 561 = 3*11*17, it is revealed to be composite using base 3 (among many others). Indeed, $3^{560} \equiv 375 \bmod 561$. As some influential researchers (e.g., Ramachandran) have proposed testing the Fermat test for arithmetical savants, this proviso is not trivial but important. Researchers could have conducted a wrong experiment! Also note that speculations by Sacks [9] concerning modular arithmetic are irrelevant (see [3]).

These criticisms should not be interpreted as personal attacks. Their errors should simply be corrected, and their lack of accurate knowledge in a certain field does not devalue their other lines of works. However, also note that some scientists may say it is unethical to copy others' texts without understanding them.

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